KNT/KW/16/5201

Bachelor of Science (B.Sc.) Semester — VI (C.B.S.) Examination

MATHEMATICS (Special Theory of Relativity)

Optional Paper—2

Time : Three Hours] [Maximum Marks : 60

N.B. : —(1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question No. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Define an inertial frame of reference. Show that the Newton's kinematical equations of motion are invariant under Galilean transformations. 6

(B) Show that the three dimensional volume element dx dy dz is not Lorentz invariant but the four dimensional volume elements dx dy dz dt is Lorentz invariant. 6

OR

(C) Prove that $\nabla^2 - \frac{1}{c^2} \cdot \frac{\partial^2}{\partial t^2}$ is invariant under special Lorentz transformations. 6

(D) Show that Lorentz transformation is simply a rotation in the four dimensional space. 6

UNIT—II

2. (A) Obtain the transformation equations for the acceleration of a particle. 6

(B) Explain the phenomenon of 'time dilation' in special theory of relativity. 6

OR

(C) Obtain the transformation of Lorentz cotraction factor $\left(1 - \frac{u^2}{c^2}\right)^{1/2}$ in the two inertial frames of references. 6

(D) Deduce the Einstein's velocity addition law from the transformation of particle velocities and prove that the velocity of light is the maximum range of velocity attainable in nature. 6

UNIT—III

3. (A) Define symmetric and skew symmetric covariant tensors of order 2. Show that any tensor of the second order may be expressed as the sum of a symmetric tensor and skew symmetric tensor. 6

(B) Let A^{pq}_{rst} be a tensor. If p = t, q = s, then show that A^{pq}_{rqp} where the summation convention is employed, is a tensor. What is its rank ? 6

OR

(C) Define four tensor and show that

$$T'^{11} = a^2\left\{T^{11} - \frac{v}{c}T^{14} - \frac{v}{c}T^{41} + \frac{v^2}{c^2}T^{44}\right\}.$$ 6

(D) Define timelike, spacelike and lightlike intervals. Prove that there exist an inertial system S´ in which the two events occur at one and the same time if the interval between two events is spacelike. 6

UNIT—IV

4. (A) Prove that $m = \dfrac{m_o}{\left(1-\dfrac{u^2}{c^2}\right)^{1/2}}$, where u is velocity of body when its mass is m and m_o is the rest mass of the body. 6

(B) From the relativistic concept of mass and energy, show that the kinetic energy of the mass m moving with velocity v is $\frac{1}{2}m_o v^2$, where $v << c$ and m_o is the rest mass of the body. 6

OR

(C) Formulate energy momentum four vector in component form in the space-time of Special Theory of Relativity. 6

(D) State the Maxwell's equations of Electromagnetic theory in vacuum. Show that the electromagnetic field strengths $\bar{E}$ and $\bar{H}$ both propagate in vacuum with velocity of light. 6

Question—V

5. (A) State the postulates on which special theory of relativity is based. 1½

(B) Discuss the outcome of Michelson-Morley experiment regarding fringe shift and stationary ether. 1½

(C) Is that the 'simultaneity' absolute in special relativity ? Explain. 1½

(D) Two electrons move towards each other with equal speed 0.8 c. Find their speed relative to each other. 1½

(E) Define : world line and proper time. 1½

(F) Prove that kronecker delta is an invariant tensor. 1½

(G) Define four velocity and four acceleration of a particle. 1½

(H) Prove that the energy momentum tensor T^{ij} is symmetric. 1½

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Bachelor of Science (B.Sc.) Semester—VI (C.B.S.) Examination

MATHEMATICS

(M_{12}: Discrete Mathematics and Elementary Number Theory)

Optional Paper—2

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Let (L, ≤) be a Lattice. For any a, b, c ∈ L, prove that :

$$b \le c \Rightarrow \begin{cases} a * b \le a * c \\ a \oplus b \le a \oplus c \end{cases}$$ 6

(B) Show that every chain is a distributive Lattice. 6

OR

(C) Prove the following Boolean identities :

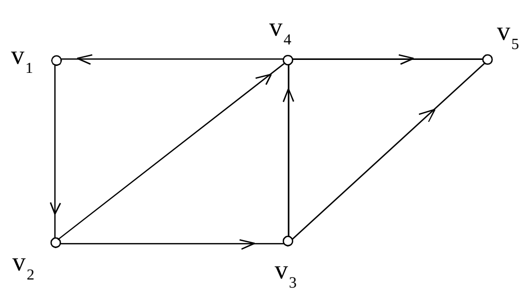
(i) $a \oplus (a' * b) = a \oplus b$

(ii) $a * (a' \oplus b) = a * b$

(iii) $(a * b) \oplus (a * b') = a$

(iv) $(a * b * c) \oplus (a * b) = a * b$. 6

(D) Find all the indegrees and the outdegrees of the digraph given below. Also find all the elementary cycles of this digraph :



6

UNIT—II

2. (A) Prove that if g is the greatest common divisor of b and c then there exist integers x_o and y_o such that $g = (b, c) = bx_o + cy_o$. 6

(B) Find the greatest common divisor and the least common multiple of 482 and 1687. 6

OR

(C) Prove that if p is prime then :

$$(p - 1)! \equiv -1 \pmod{p}.$$ 6

(D) Solve the linear congruence :

$$6x \equiv 15 \pmod{21}.$$ 6

UNIT—III

3. (A) Let p be an odd prime and let a and b be integers relatively prime to p i.e. (a, b) = 1 and (b, p) =1. Then prove that :

$$\left(\frac{a}{p}\right)\left(\frac{b}{p}\right) \equiv \left(\frac{ab}{p}\right)$$ 6

(B) Determine which of the following congruence are solvable :

(i) $x^2 \equiv 5 \pmod{227}$

(ii) $x^2 \equiv -5 \pmod{227}$. 6

OR

(C) Define Jacobi symbol $\left(\frac{P}{Q}\right)$, and prove that :

(i) $\left(\frac{P}{Q}\right)\left(\frac{P}{Q'}\right) = \left(\frac{P}{QQ'}\right)$

(ii) $\left(\frac{P}{Q}\right)\left(\frac{P'}{Q}\right) = \left(\frac{PP'}{Q}\right)$ 6

(D) If p is an odd positive prime integer, then prove that :

$$\left(\frac{-1}{p}\right) = \begin{cases} 1, & \text{if } p \equiv 1 \pmod{4} \\ -1, & \text{if } p \equiv -1 \pmod{4} \end{cases}$$ 6

UNIT—IV

4. (A) Find all positive solutions of Diophantine equation $5x + 3y = 52$. 6

(B) If <x, y, z> is a primitive Pythagorean triplet, then prove that x and y are of opposite parity. 6

OR

(C) Find all the primitive solutions of $x^2 + y^2 = z^2$ with the condition $0 < z < 30$. 6

(D) If $\frac{a}{b}$ and $\frac{a'}{b'}$ are two consecutive terms in a Farey sequence with $\frac{a}{b}$ to the left of $\frac{a'}{b'}$, then prove that $a'b - ab' = 1$. 6

Question—V

5. (A) Draw Hasse-diagram of the set P = {2, 3, 6, 12, 24, 36} with a partial order relation 'divide' in P. 1½

(B) Define a complete Lattice. 1½

(C) Prove that for $a, b, c \in Z$, $a|b$ and $b|c \Rightarrow a|c$. 1½

(D) Prove that $a \equiv b \pmod{m} \Leftrightarrow b \equiv a \pmod{m}$. 1½

(E) Show that 4 is quadratic residue (qr) of 7, but 3 is quadratic non residue (qnr) of 7. 1½

(F) Show that $\left(\frac{22}{105}\right) = -1$. 1½

(G) Define primitive Pythagorean triplet with an example. 1½

(H) Prove that terms in a Farey Sequence are in monotonically increasing order. 1½

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Bachelor of Science (B.Sc.) Semester-VI (C.B.S.) Examination

MATHEMATICS (M_{12} Differential Geometry)

Optional Paper—2

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Questions 1 to 4 have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Show that oscillating plane at any point P (x_p, y_p, z_p) is given by

$$\begin{vmatrix} x - x_p & y - y_p & z - z_p \\ \dot{x}_p & \dot{y}_p & \dot{z}_p \\ \ddot{x}_p & \ddot{y}_p & \ddot{z}_p \end{vmatrix} = 0$$ 6

(B) State and prove Frenet Serret formulae. For the space curve $\bar{r} = \bar{r}(s)$; where s is the length of the arc of a curve measured from a fixed point A on it to a point P on it. 6

OR

(C) Prove necessary and sufficient condition for a curve to be a helix is that T/K is constant. 6

(D) Find $\bar{t}$, $\bar{n}$, $\bar{b}$ for the twisted curve $x = 3t$, $y = 3t^2$, $z = 2t^3$ at a point $t = 1$. 6

UNIT—II

2. (A) Find the equation of the involute of the circular helix.

$\bar{r}$ = [a cos t, a sin t, bt]. 6

(B) Prove that the length of the evolute is equal to the difference in the values of the raddii of curvature at the end points on the curve C. 6

OR

(C) Find the envelopes of the plane

$[x/(a + u)] + [y (b + u)] + [z/(c + u)] = 1$

where u is the parameter, and determine the edge of regression. 6

(D) Show that the line given by $y = tx - t^2$, $z = t^3 y - t^6$ generates a developable surface. 6

UNIT—III

3. (A) Derive :

(i) The First Fundamental form

$ds^2 = E\,du^2 + 2F\,dudv + G\,dv^2$,

where $E = \bar{r}_1^2, F = \bar{r}_1 \circ \bar{r}_2, G = \bar{r}_2^{2'}$ and

(ii) The second fundamental form

$L\,du^2 + 2M\,dudv + N\,dv^2$,

where $L = \bar{N} \circ \bar{r}_{11}, M = \bar{N} \circ \bar{r}_{12}, N = \bar{N} \circ \bar{r}_{22}$. 6

(B) Taking x, y as parameters, calculate the fundamental magnitudes and the normal to the surface $2z = ax^2 + 2hxy + by^2$. 6

OR

(C) Obtain Gauss's formulae for $\bar{r}_{11}, \bar{r}_{12}, \bar{r}_{22}$ where $\bar{r}$ is the position vector of any point on a surface and suffixes 1 and 2 denotes differentiation with regard to u and v respectively. 6

(D) Show that the curvature K at any point P of the curve of intersection of two surfaces is given by $K^2 \sin^2 \alpha = K_1^2 + k_2^2 - 2K_1K_2 \cos \alpha$, where K_1 and K_2 are normal curvatures of the surfaces in the direction of the curve at P, and α is the angle between their normals at that point. 6

UNIT—IV

4. (A) Prove that the curves of the family $v^3 = cu^2$ are geodesics on a surface with metric :

$v^2\,du^2 - 2uv\,dudv + 2u^2\,dv^2$,

$(u > 0, v > 0)$. 6

(B) Obtain the differential equation of geodesic on a surface of revolution $Z = f\left(\sqrt{x^2 + y^2}\right)$ and deduce that on a right cylinder the geodesics are helices. 6

OR

(C) If the Gaussian curvature K of a surface is continuous on a simply connected region R enclosed by a closed curve C, composed of n smooth arcs making at the vertices exterior angles $\alpha_1, \alpha_2, \ldots, \alpha_n$ then prove

$$\iint_R K\,dS = 2\pi - \sum_{i=1}^{n} \alpha_i - \int_e K_g\,dS$$

where K_g represents the geodesic curvature of the arcs, dS is the element of area of S. 6

(D) If two families of geodesics on a surface intersects at a constant angle, prove that the surface has zero Gaussian Curvature. 6

Question—V

5. (A) Define curvature and torsion of the curve. 1½

(B) Define Helix. 1½

(C) State fundamental theorem of space curves. 1½

(D) Prove that the Locus of the centre of curvature is an evolute only when the curve is plane. 1½

(E) Define Gaussian curvature of a surface at any point P. 1½

(F) Prove that, if θ is the angle at the point (u, v) between the two directions :

$$Pdu^2 + 2Q\,dudv + R\,dv^2 = \theta, \text{ then}$$

$$\tan\theta = 2H^2 (Q^2 - PR)^{1/2}/(ER - 2FQ + GP).$$ 1½

(G) Write cannonical equations for geodesics. 1½

(H) Define geodesic polar coordinates for the geodesic metric $ds^2 = du^2 + G(u, v)\,dv^2$. 1½